Enrollment No. ____________

Shree Manibhai Virani and Smt. Navalben Virani Science College (Autonomous)

Affiliated to Saurashtra University, Rajkot

SEMESTER END EXAMINATION APRIL – 2017

M. Sc. Mathematics

16PMTCC07 - TOPOLOGY - II

Duration of Exam – 3 hrs ***Semester – II*** ***Max. Marks – 70***

***Part A* (5x2= 10 marks)**

Answer **ALL** questions

1. Let X be the subspace [-1, 1] of the real line. The set [-1, 0] and (0, 1] . form a separation of X. Is this statement TRUE? Justify.

2. Define: i) Subnet ii) A Free filter.

3. Specify base for the box topology on a collection $\{(X_{\sim}, {}^2{}_{\sim}) / \sim \in \Lambda\}$ of topological spaces (with $\mathcal{T}_{\sim}$ as its topology).

4. Define: i) Covering of a space. ii) Bolzano-Weierstrass property.

5. Give an example with explanation: A topological space which is both Compact and locally compact space.

***Part B* (5X5 = 25 marks)**

Answer **ALL** questions

6a. If (X,$\mathcal{T}$) and (Y,$\mathcal{T}$) are connected topological spaces and f:(X, $\mathcal{T}$) → (Y, $\mathcal{T}'$) is a continuous map then prove that f(X) is also a connected space..

OR

6b. Prove that the collection of connected subspaces of a topological space (X,$\mathcal{T}$) that have a point in common is connected.

7a. Let (X, $\mathcal{T}$) be a topological space and x$\in$X. Suppose Λ is a fixed nbhd base at x$\in$X. Define the order $\leq$ in Λ as

$U_1 \leq U_2 \Leftrightarrow U_2 \subset U_1$ for U_1 , $U_2 \in \mathcal{U}_x = \Lambda$ then prove that

(i) The $\Lambda = \mathcal{U}_x$ along with the relation $\leq$ is a directed set (Λ ,$\leq$).

(ii) If we take some $x_U \in U$, $\forall$ U$\in \Lambda$ then $(x_U)_{U\in\Lambda}$ is a net on X and

(iii) Prove that $x_U \to x$.

OR

7b. Let (X, $\mathcal{T}$) be a topological space and x $\in$ X .

If E$\subset$X the x$\in \overline{E} \Leftrightarrow$ there is a net $(x_a)_{a\in\Lambda}$ in E with $x_a \to x$.

8a. Let (X, $\mathcal{T}$) be a topological space. Let be a set. Let p: (X, $\mathcal{T}$) $\to Y$ be an onto(surjective) map consider the collection $\Im_p = \{U \subset Y / p^{-1}(U) \in \Im\}$ then prove that $\Im_p$ is a topology

Such that p: (X, $\mathcal{T}$) $\to Y$ is continuous.

OR

8b. Define: Quotient mapping,

Let X=R with usual topology T_U on it, Y = {a, b, c}

p(x) = a if x>0,

= c if x = 0

= b if x < 0 then prove that

i) p is a quotient map.

ii) Specify what is the quotient topology on it.

9a. Let (X, $\mathcal{T}$) and (Y,$\mathcal{T}'$) be topological spaces. If $x_0 \in$ X and Y is compact then $x_0 \times$ Y is compact subspace of X×Y.

OR

9b. Let (X, $\mathcal{T}$) be a Hausdorff topological space, Let $Y \subset X$ be compact subspace of X. Let $x_0 \in X$ be such that $x_0 \notin Y$ then there exist $U, V \in \Im$ such that $U \cap V =$ ¤, $x_0 \in U$ and $Y \subset V$.

10a. Let (X, $\mathcal{T}$) be a Hausdorff topological space, then X is locally compact iff for every x in X and for every nbhd U of x and there exist a nbhd V of x such that $\overline{V}$ is compact and $\overline{V} \subset U$ $\{i.e\ \ x \in V \subset \overline{V} \subset U\}$

OR

10b. Prove that compactness implies limit point compactness.

<u>***Part C***</u> ***(5X7 = 35 marks)***

Answer <u>**ALL**</u> questions

11a. Define: Locally Path Connected Space at a point.

Let (X, $\mathcal{T}$) be a topological space. Let (A,$\mathcal{T}_A$) be a connected subspace of X and let B be a subset of X such that $A \subset B$ and $B \subset \overline{A}$ (i.e. $A \subset B \subset \overline{A}$) then prove that (B, T_B) is connected.

OR

11b. Define: Path Component.

Let (X, $\mathcal{T}$) be a connected topological space and let (Y, $\mathcal{T}'$) be a simply ordered set equipped with the usual ordered topology $\mathcal{T}'$ on Y. Let f:(X, $\mathcal{T}$) → (Y, $\mathcal{T}'$) be a continuous function then prove that, if a, b $\in$ X, and r $\in$ Y are ∋ f(a) < r < f(b) then ∃ a point $c \in X \ni f(c) = r$.

12a. If (X, $\mathcal{T}$) is a topological space and if $(x_a)_{a\in\Lambda}$ is an ultra net in X, and $f : X \to Y$ then $(f(x_a))_{a\in\Lambda}$ is an ultra net in Y. Describe and justify any two properties of a filter.

OR

12b. Suppose (X, $\mathcal{T}$) and (Y, $\mathcal{T}$) are topological Space, then $f : X \to Y$ is continuous mapping at $x_0 \in X$, then

$f : X \to Y$ is continuous at $x_0 \in X \leftrightarrow \mathcal{F} \to x_0$ then $f(\mathcal{F}) \to x_0$

13a. Define: Quotient Topology.

Let (X, $\mathcal{T}$) be a topological space. Let Y be any set, then prove that p is a quotient mapping if the following condition holds,

"A is subset of Y is closed iff and p^{-1}(A) is closed."

OR

13b. Define: Product topology on $X_1 \times X_2 \times X_3 \times .... \times X_n$.

Let X = RxR and Y = R and let $\prod_1 : R \times R \to R \quad for$ every $(x, y) \in R \times R$ then prove that $\prod_1$ is continuous, open, and surjective mapping.

14a. Define: Compact Topological Space.

Let A and B be disjoint Compact subsets of a Housdorff space (X, $\mathcal{T}$) then prove that there are disjoint open sets U and V such that $A \subset U \ and \ B \subset V$.

OR

14b. If (X, $\mathcal{T}$) and (Y, $\mathcal{T}$) are compact topological Space then prove that product $X \times Y$ is also a compact topological space.

15a. Let (X, T) be a topological space.

Assume that the space X is locally compact Hausdorff space iff there exist space Y satisfying following conditions,

(i) X is a subspace of Y.

(ii) The set Y-X consists of a single point.

(iii) Y is a compact Hausdorff topological space.

Only prove the following

(1) X is a subspace of Y. (2) Y is a Hausdorff space.

OR

15b. Define: Compactification.

Let (X, $\mathcal{T}$) be a topological space then prove that X is compact iff for every collection $\mathcal{A} = \{A_\alpha / \alpha \in J\}$ of closed subsets of X with finite intersection property then $\bigcap_{\alpha \in J} A_\alpha \neq \phi$.
